The University of Texas at Austin
Hildebrand Department of Petroleum and Geosystems Engineering
Cockrell School of Engineering

Reservoir Engineering – Primary Recovery

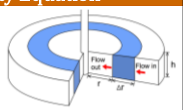
Conversions and Constants

1 kPa = 0.1450 psi 1 MPa = 10 bar 1 atm = 14.696 psi 1 atm = 1.013 bar	1 in = 2.54 cm 1 ft = 0.3048 m 1 mile = 5,280 ft	1 acre = 43,560 ft^2 1 m^2 = 10.764 ft^2	1 m^3 = 6.2898 bbl 1 bbl = 5.6146 ft^3 1 bbl = 42 US gal 1 ft^3 = 7.4805 gal	R = 459.67 + °F K = 273.15 + °C °F = 1.8 °C + 32	1 cp = 1.0 mPa-s 1 lb$_m$ = 453.592 g

1 Newton = 1 x 10^5 dynes	1 Darcy = 9.8692 x 10^{-9} cm^2	Standard Temperature = 60°F
1 dyne = 2.248 x 10^{-6} lb$_f$	1 Darcy = 1.0623 x 10^{-11} ft^2	Standard Pressure = 14.696 psia
1 g/cm^3 = 62.428 lb$_m$/ft^3	Euler (γ) = 0.5772 = ln(1.781)	Water density at SC = 62.37 lb$_m$/ft^3
1 mD/cp = 6.33 x 10^{-3} ft^2/psi-day	Gravitational Constant = 9.806 m/s^2	Molar Mass of Air = 28.966 g/mol
1000 kg/m^3 = 0.4335 psi/ft	Natural logarithm base = 2.71828	V_M^{IG} = 379.3 scf/lbmol @ 14.696 psia
1 kg/l = 8.347 lb$_m$/gal	Gas Constant = 8.314 Pa · m^3/mol K	Univ. Gas = 10.732 psia · ft^3/lbmol · R

Diffusivity Equation

Mass Balance leads to Continuity Equation



$$-\nabla(\rho u) = \frac{\partial(\rho\phi)}{\partial t}$$

$$-\frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} = \frac{\partial(\rho\phi)}{\partial t} \qquad -\frac{\partial(\rho u_x)}{\partial x} = \frac{\partial(\rho\phi)}{\partial t}$$

Introduce Darcy's Law $\quad u_r = -\frac{k}{\mu}\nabla P = -\frac{k}{\mu}\frac{\partial P}{\partial r}$

Introduce Formation Volume Factor $\quad B = \frac{\rho_{SC}}{\rho_{RC}}$

Introduce Compressibility $\quad c_t = c_f + c_{fluid}$

$$c_f = \frac{1}{\phi}\left(\frac{\partial\phi}{\partial P}\right)_T \qquad c_{fluid} = B\frac{\partial}{\partial P}\left(\frac{1}{B}\right)$$

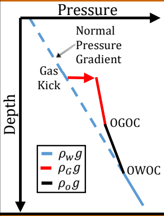
Introduce Diffusivity Constant $\quad \alpha = \frac{k}{\mu\phi c_t}$

1D Diffusivity Equation

$$\frac{\partial P}{\partial t} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) \qquad \frac{\partial P}{\partial t} = \alpha\frac{\partial^2 P}{\partial x^2}$$

$$\frac{\partial P}{\partial t} = \alpha(\nabla^2 P)$$

Reservoir Pressure and Temperature



$$P = P_{surface} + \alpha_P z$$

z = depth $\quad \alpha_P = \rho_f g$

$$\alpha_P = \begin{cases} 0.433\ \frac{psi}{ft} & \text{if fresh } H_2O \\ 0.465\ \frac{psi}{ft} & \text{if brine} \end{cases}$$

$$T = T_{surface} + \alpha_T z$$

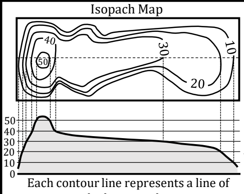
α_T is usually 0.01 – .02°F/ft

Volumetrics

$V_p = Ah\phi \qquad N = \frac{V_p S_o}{B_o}[=]STB \qquad G = \frac{V_p S_g}{B_g}[=]scf$

V_p = reservoir pore volume $\qquad S_o$ = average oil saturation

Isopach Map

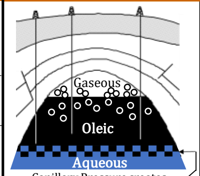


$$V_1 = h\left(\frac{A_0 + A_{10}}{2}\right) \qquad V_2 = h\left(\frac{A_{10} + A_{20}}{2}\right)$$

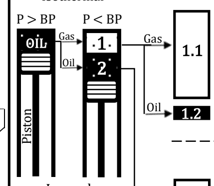
$$N = \sum_{n=1}^{5}\frac{V_n\phi_n S_{o_n}}{B_o}$$

Each contour line represents a line of constant thickness in the reservoir

Reservoir Fluids



Capillary Pressure creates transition zone between phases



Legend
1.1 – Free Gas
1.2 – Volatilized Oil
2.1 – Solution Gas
2.2 – Free Oil
1 – Gaseous Phase (Gas + Volatilized Oil)
2 – Oleic Phase (Oil + Solution Gas)

$$B_{tg} = \frac{B_g(1 - R_{vi}R_s) + B_o(R_{vi} - R_v)}{1 - R_s R_v}$$

$$B_g = \frac{\boxed{1}}{1.1} \qquad R_v = \frac{1.2}{1.1} \qquad B_{tg} = \frac{\boxed{1} + \boxed{2}}{1.1 + 2.1}[=]\frac{RB}{scf}$$

$$B_{to} = \frac{B_o(1 - R_{si}R_v) + B_g(R_{si} - R_s)}{1 - R_s R_v}$$

$$B_o = \frac{\boxed{2}}{2.2} \qquad R_s = \frac{2.1}{2.2} \qquad B_{to} = \frac{\boxed{1} + \boxed{2}}{1.2 + 2.2}[=]\frac{RB}{STB}$$

$B_o[=]RB/STB \qquad R_s[=]scf/STB \qquad B_g[=]rcf/scf \qquad R_v[=]STB/scf$

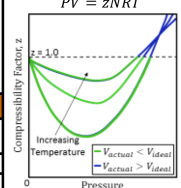
Component Concentrations

$$C_o = \frac{S_o}{B_o} + \frac{S_g}{B_g}R_v \qquad C_w = \frac{S_w}{B_w}$$

$$C_g = \frac{S_g}{B_g} + \frac{S_o}{B_o}R_s \qquad C_j[=]\frac{STB}{RB}$$

Compressibility Factor

$$PV = zNRT$$

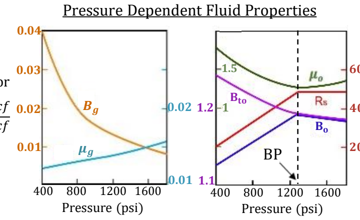


Relate B_g to z-factor

$$B_g = \frac{P^{SC}}{T^{SC}}\left(\frac{zT}{P}\right)[=]\frac{rcf}{scf}$$

$P[=]psia \quad T[=]R$

Pressure Dependent Fluid Properties



Specific Gravity

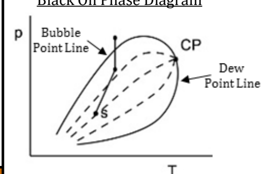
$$\gamma_o = \left(\frac{\rho_o}{\rho_w}\right)_{SC}$$

$$\gamma_g = \left(\frac{\rho_g}{\rho_{air}}\right)_{SC} = \frac{M_g}{M_{air}}$$

API Gravity

$$API^o = \frac{141.5}{\gamma} - 131.5$$

Black Oil Phase Diagram



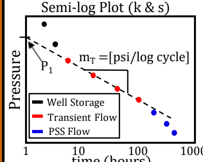
Undersaturated Reservoir Key Properties

Fluid Type	G_{fgi}	N_{foi}	G_p	N_p	R_v	R_s
Dry Gas	>0	0	>0	0	0	--
Wet Gas	>0	0	>0	>0	R_{vi}	0
Condensate	>0	0	>0	>0	>0	0
Volatile Oil	0	>0	>0	>0	>0	>0
Black Oil	0	>0	>0	>0	0	>0
Undersaturated Oil	0	>0	>0	>0	0	R_{si}
Dead Oil	0	>0	0	>0	--	0

Well Testing

Drawdown Test Analysis

Semi-log Plot (k & s)

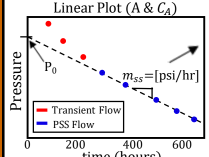


$$k = 162.6\left|\frac{q_{sc}B_o\mu}{hm_T}\right|$$

$$s = 1.151\left[\frac{P_i - P_1}{|m_T|} - \log\frac{k_o}{\mu_o c_t r_w^2\phi} + 3.23\right]$$

$k[=]md$	$q_{sc}[=]$STB/day	$h[=]ft$	$\mu[=]cp$
$c_t[=]psi^{-1}$	$P[=]psi$		$r_w[=]ft$

Linear Plot (A & C_A)



$$A = \left|\frac{q_{sc}B_o}{c_t h\phi m_{ss}}\right|[=]\text{consistent units}$$

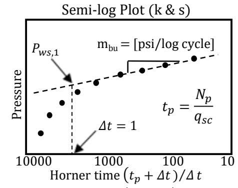
$$\log C_A = \log\frac{4A}{1.781 r_w^2} + 0.87s + \frac{P_0 - P_i}{|m_T|}$$

$A, r_w^2[=]ft^2$ $\quad$ **mT from semi-log plot

$C_A = 31.62$ (circle)	1:4 $C_A = 5.38$
$C_A = 30.88$ (square)	1:2 $C_A = 22.6$

Buildup Test Analysis

Semi-log Plot (k & s)



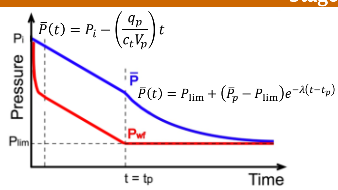
$$k = 162.6\left|\frac{q_{sc}B_o\mu}{hm_{bu}}\right|$$

$$P_{ws} = P_i - \frac{162.6B|q_{sc}|\mu}{kh}\log\left(\frac{t_p + \Delta t}{\Delta t}\right)$$

$$s = 1.151\left[\frac{P_{ws,1} - P_{wf}}{|m_{bu}|} - \log\frac{k_o}{\mu_o c_t r_w^2\phi} + 3.23\right]$$

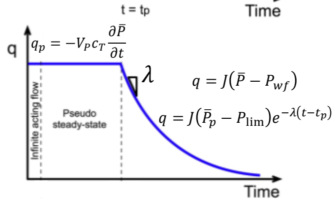
$h[=]ft$	$t_p[=]hr$	$P[=]psi$	$q_{sc}[=]$STB/day
$r_w[=]ft$	$\mu[=]cp$	$k[=]md$	$c_t[=]psi^{-1}$

Stages of Flow



$$\bar{P}(t) = P_i - \left(\frac{q_p}{c_t V_p}\right)t$$

$$\bar{P}(t) = P_{lim} + (\bar{P}_p - P_{lim})e^{-\lambda(t - t_p)}$$

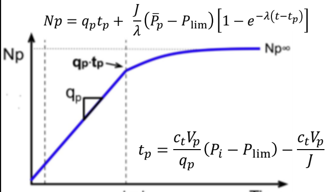


$$q_p = -V_p c_T\frac{\partial\bar{P}}{\partial t}$$

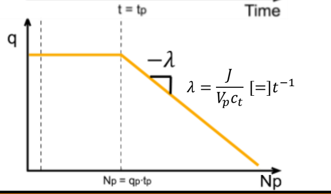
$$q = J(\bar{P} - P_{wf})$$

$$q = J(\bar{P}_p - P_{lim})e^{-\lambda(t - t_p)}$$

$$Np = q_p t_p + \frac{J}{\lambda}(\bar{P}_p - P_{lim})\left[1 - e^{-\lambda(t - t_p)}\right]$$



$$t_p = \frac{c_t V_p}{q_p}(P_i - P_{lim}) - \frac{c_t V_p}{J}$$



$$\lambda = \frac{J}{V_p c_t}[=]t^{-1}$$

1. Infinite Acting/Transient Flow

$$P(r,t) = P_1 + \frac{q\mu}{4\pi kh}Ei\left(-\frac{\phi\mu c_T r^2}{4kt}\right)$$

$$Ei(-x) = \ln(x) + \gamma - \sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n(n!)}$$

$E_i(-x) \approx \ln(e^\gamma x)$ Valid if $x < 0.01$

$$P_{wf}(t) \approx P_i - \frac{162.6q_{sc}B\mu}{kh}\left[\log(t) + \log\left(\frac{k}{\mu\phi c_t r_w^2}\right) - 3.23\right]$$

Field Units

2. Transitional/Late Transient Flow

- When pressure disturbance first approaches boundary
- Short time period and difficult to model mathematically

3. Stabilized Flow

Pseudo/Semi SS	Actual Performance	Steady State (SS)
Closed drainage boundary $f = 0$	$0 < f < 1$	Open drainage boundary $f = 1$

$$q_{osc} = \frac{2\pi kh(\bar{P} - P_w)}{\mu_o B_o\left(\ln\frac{r_e}{r_w} - \frac{1}{2} - \frac{(1 - f)}{4} + s\right)}$$

**For gas flow, replace pressure with pseudopressure:

$$m(P) = \int_{P_{ref}}^{P}\frac{2P}{\mu z}\partial P$$

$$P_1 - P_2 = \frac{q\mu}{2\pi kh}\left\{\ln\frac{r_1}{r_2} - \frac{(1 - f)}{2}\left[\left(\frac{r_1}{r_e}\right)^2 - \left(\frac{r_2}{r_e}\right)^2\right]\right\}$$

$$\text{PSS:}\ P_{wf}(t) = P_i - \frac{q_{sc}\mu B}{2\pi kh}\left\{\frac{1}{2}\left(\ln\frac{4A}{1.781C_A r_w^2}\right) + \frac{2\pi kt}{\phi\mu c_T A} + s\right\}$$

4. Depletion Flow

- P_{wf} limit reached; well's BHP held constant
- Well's flow rate begins to decline
  - Exponential decline can be derived (equations shown to the left)
  - Hyperbolic b-factor is empirical (refer to Decline Curve Analysis)

Material Balance Equation

$$\underbrace{G_{fgi}E_g}_{\text{Free-gas expansion}} + \underbrace{N_{foi}E_o}_{\text{Free-oil expansion}} + \underbrace{WE_w}_{\text{Free-water expansion}} + \underbrace{V_{pi}E_f}_{\text{Rock expansion}} + \underbrace{W_e}_{\text{Water influx}} = \underbrace{(G_P - G_I)\left(\frac{B_g - B_o R_v}{1 - R_v R_s}\right)}_{\text{Net gas withdrawal}} + \underbrace{N_P\left(\frac{B_o - B_g R_s}{1 - R_v R_s}\right)}_{\text{Net oil withdrawal}} + \underbrace{(W_P - W_I)B_w}_{\text{Net water withdrawal}}$$

G_{fgi} = Original free gas[=]scf $\quad N_{foi}$ = Original free oil[=]STB $\quad W_e[=]RB \quad G_P$ = Cumulative Gas Produced[=]scf $\quad G_I$ = Gas Injected[=]scf

$$W = \frac{V_{pi}S_{wi}}{B_{wi}} \qquad E_w = B_{wi}c_w\Delta P \qquad E_f = \frac{V_{pi} - V_p}{V_{pi}} \approx c_f\Delta P \qquad c_f \approx c_p = \frac{1}{\phi}\left(\frac{\partial\phi}{\partial P}\right)_T$$

Can combine free-water expansion and rock expansion into composite expansivity

$$E_{owf} = E_o + B_{oi}c_T\Delta P \qquad E_{gwf} = E_g + B_{gi}c_T\Delta P \qquad E_{owf} \approx E_o$$

$$E_o = B_{to} - B_{toi} \qquad E_g = B_{tg} - B_{tgi} \qquad E_{gwf} \approx E_g$$

$$c_T = \frac{c_f + c_w S_{wi}}{1 - S_{wi}} \qquad \text{*if } c_w \text{ and } c_f \text{ are neglected}$$

$$G_{fgi}E_{gwf} + N_{foi}E_{owf} + W_e = F = (G_p - G_I)\left(\frac{B_g - B_o R_v}{1 - R_v R_s}\right) + N_p\left(\frac{B_o - B_g R_s}{1 - R_v R_s}\right) + (W_P - W_I)B_w$$

$E_{gwf}[=]RB/scf \quad E_{owf}[=]RB/STB \quad F$ = Reservoir Voidage[=]RB $\quad N_p$ = Cumulative Oil Produced[=]STB

Aquifer Models

Pot Aquifer (Coats, 1970)
- Assumes instantaneous response
- No time dimension

$$W_e = W'(c_f + c_w)\Delta P$$

Fetkovitch (1971)
- Assumes SSS relationship
- Not applicable for transient response

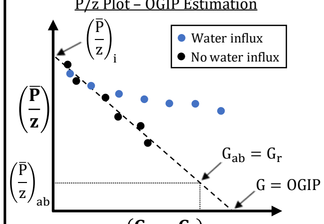
$$\frac{dW_e}{dt} = J(P_a - \bar{P})$$

Hurst and Van Everdingen (1949)
- Constant terminal-pressure solution
- Time based (superposition principle)
- Applicable for transient and SSS flow

$$W_e = B\sum\Delta PQ_D(t_D)$$

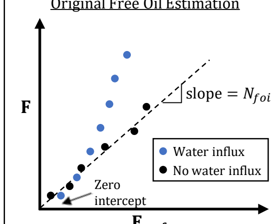
Carter-Tracy (1960)
- Constant terminal-rate solution
- Incorporates transient and SSS flow
- Preferred model for simulators

P/z Plot – OGIP Estimation



$$\left(\frac{\bar{P}}{z}\right) = \underbrace{\left(\frac{\bar{P}}{z}\right)_i}_{\text{intercept}} - \underbrace{\left[\left(\frac{\bar{P}}{z}\right)_i\frac{1}{G}\right]}_{\text{slope}}(G_P - G_I) \qquad \underbrace{RF = \frac{G_r}{G}}_{\text{Recovery factor}}$$

Original Free Oil Estimation



$$F = N_{foi}E_{owf} \quad (G_{fgi} = 0)$$

Plot applicable for undersaturated reservoirs with Volatile, Black, Undersaturated, or Dead Oils

Drive Indices

$\frac{G_{fgi}E_g}{F} = I_{gd}$ Gas cap drive

$\frac{N_{foi}E_o}{F} = I_{sd}$ Solution gas drive

$\frac{WE_w}{F} + \frac{V_{pi}E_f}{F} = I_{fwd}$ Formation drive

$\frac{W_e}{F} = I_{nwd}$ Natural water drive

Drive Indices must add up to 1.0
**Different definition can be used to account for an artificial water drive

Productivity Index

$$J = \frac{q}{(\bar{P} - P_{wf})}[=]\frac{volume}{psi \cdot day}$$

$$J = \frac{2\pi k_j h}{\mu_j B_j\left[\ln\left(\frac{r_e}{r_w}\right) + s - \frac{1}{2} - \frac{(1 - f)}{4}\right]}$$

$$J = \frac{2\pi k_j h}{\mu_j B_j\left[\frac{1}{2}\ln\left(\frac{4A}{1.781C_A r_w^2}\right) + s\right]}$$

J is constant after transient flow period

Formation Skin

$(+)Skin\ q\downarrow \qquad \Delta P_{skin} = \frac{q_{sc}B\mu}{2\pi kh}s$

$(-)Skin\ q\uparrow \qquad \Delta P_{skin} = 0.869|m|s$

m = transient slope

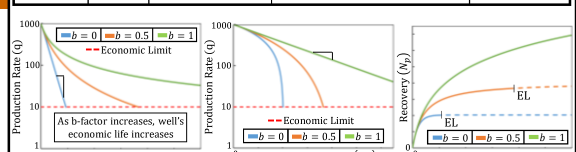
$$\frac{q_{stim}}{q_{orig}} = \frac{J_{stim}}{J_{orig}} = \frac{\ln(r_e/r_w) + s_{orig}}{\ln(r_e/r_w) + s_{stim}}$$

Skin is unitless and specific to each well

Decline Curve Analysis

$$\text{ODE: } -\frac{1}{q}\frac{dq}{dt} = D_i\left(\frac{q}{q_i}\right)^b;\ q = q_i\ @\ t = 0;\ \frac{D_i}{q_i^b}\int_{t=0}^{t}dt = -\int_{q_i}^{q}\frac{dq}{q^{b+1}} \qquad N_p = \int_{t=0}^{t}q(t)dt$$

(Arps, 1940)	b-factor	Rate (q)	Cumulative Recovery (N_p)	Decline Rate (D)
Exponential	$b = 0$	$q = q_i e^{-D_i t}$	$N_p = \frac{(q_i - q)}{D_i}$	$D = D_i$
Hyperbolic	$0 < b < 1$	$q = \frac{q_i}{(1 + bD_i t)^{1/b}}$	$N_p = \frac{q_i^b}{D_i(1 - b)}(q_i^{1-b} - q^{1-b})$	$D = D_i\left(\frac{q}{q_i}\right)^b$
Harmonic	$b = 1$	$q = \frac{q_i}{(1 + D_i t)}$	$N_p = \frac{q_i}{D_i}\ln\left(\frac{q_i}{q}\right)$	$D = D_i\left(\frac{q}{q_i}\right)$



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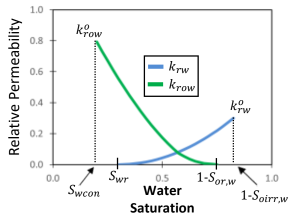
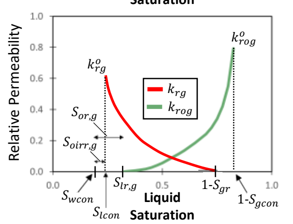
Reservoir Engineering – Improved Recovery

Petrophysics Review





Relative Permeability

$k_j = kk_{rj}$

Water Wet Rock

$k^o_{rw} < k^o_{ro}$

$S_w > 0.5$ when k_{rw} curve intersects k_{ro} curve

*Oil-Wet Rock is inverse

3-Phase Permeability

Water Wet Rock

k_{rw} is a function of S_w

$k_{ro}(S_o, S_w)$ $k_{rg}(S_g)$

Oil Wet Rock

$k_{rw}(S_w, S_o)$

$k_{ro}(S_o)$ $k_{rg}(S_g)$

Mixed-Wet Rock

$k_{ro}(S_o, S_w)$

$k_{rw}(S_w, S_o)$

$k_{rg}(S_g)$

Capillary Pressure

$$P_c = \frac{2\sigma\cos\theta}{r}$$

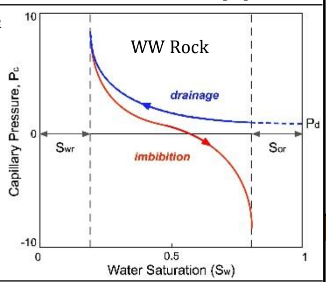
$$P_c(S_w) = P_{nw} - P_w$$

nw = non-wetting phase

$$P_c(z) = \Delta\rho_f gz$$

$$J(S_w) = \frac{P_c}{\sigma}\sqrt{\frac{k}{\phi}}$$

$$\frac{P_{c1}}{P_{c2}} = \frac{\sqrt{k_2/\phi_2}}{\sqrt{k_1/\phi_1}}$$



Permeability Variations

$$\bar{k} = \frac{\sum L_i}{\sum \frac{L_i}{k_i}}$$ Horizontal Variation

$$\bar{k} = \frac{\sum k_i h_i}{\sum h_i}$$ Vertical Variation

$$\bar{k} = \frac{\ln(r_e/r_w)}{\sum \frac{\ln(r_i/r_{i-1})}{k_i}}$$ Radial Variation

$$\bar{k} = (k_i k_{i+1} \dots k_n)^{1/n}$$ Geometric Mean

Darcy's Law Variations

$$u = -\frac{k}{\mu}\frac{\partial\Phi}{\partial x} \qquad \Phi = P + \rho gh$$

Forcheimer

(valid at high flow rates, Re>1)

$$-\frac{\partial P}{\partial x} = \frac{\mu u}{k} + \beta\rho u^2$$

β material constant; depends on pore structure

Klinkenberg Effect

(valid for gas flow at low pressure)

$$k_g = k_L\left(1 + \frac{b}{P}\right) \qquad k_L = \text{liquid perm} \quad k_g = \text{gas perm}$$

$k_L[=]k_\infty$ b factor important when k < 10md

Miscible Displacement

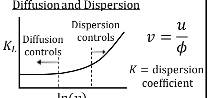
Mechanisms of Mixing

1. Molecular Diffusion D_m

$$D_e = \frac{D_m}{\tau} = \frac{D_m}{F\phi} \qquad F = \frac{a}{\phi^m}$$

τ = tortuosity

Diffusion and Dispersion



$$v = \frac{u}{\phi}$$

K = dispersion coefficient

2. Convective Mixing (Dispersion)

Longitudinal $K_L = D_e + \alpha_L v^{\beta_L}$

Transverse $K_T = D_e + \alpha_T v^{\beta_T}$

α = dispersivity $K_L >> K_T$

D_e = effective diffusivity $\beta_T \cong 1.2$

3. Small Scale Permeability Variations

Convection - Dispersion

$$\frac{\partial C_D}{\partial t_d} + \frac{\partial C_D}{\partial x_d} - \frac{1}{N_{Pe}}\frac{\partial^2 C_D}{\partial x_d^2} = 0$$

$$C_D(x_d t_d) = \frac{1}{2} erfc\left(\frac{x_d - t_d}{2\sqrt{t_d/N_{pe}}}\right)$$

$$+\frac{e^{x_d N_{Pe}}}{2} erfc\left(\frac{x_d + t_d}{2\sqrt{t_d/N_{Pe}}}\right)$$ 2nd term negligible

$$erfc(n) = 1 - erf(n)$$

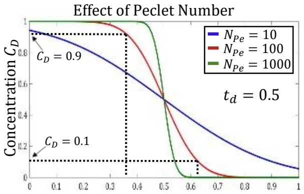
$$C_D = \frac{c - c_i}{c_{inj} - c_i} \qquad C_e = C_D|_{X_d=1}$$

$C_D = 0.5$ when $t_d = 1$

Pulse/Slug Injection (from time 0 to t_s)

$$C_D = \frac{1}{2}\left[erf\left(\frac{x - v(t - t_s)}{\sqrt{4K_L(t - t_s)}}\right) - erf\left(\frac{x - vt}{\sqrt{4K_L t}}\right)\right]$$

Effect of Peclet Number



$$N_{Pe} = \frac{uL}{\phi K_L} \cong \frac{L}{\alpha_L}$$

$$x_d = \frac{x}{L} \qquad t_d = \frac{ut}{\phi L}$$

$t_d = 0.5$

$x_d|_{C_D=0.1} - x_d|_{C_D=0.9} = \Delta x_d$

Δx_d = width of mixing zone

$$\Delta x_d = 3.625\sqrt{\frac{t_d}{N_{Pe}}}$$

Multiphase Flow

Darcy's Law for Multiple Phases

$$\tilde{u}_j = -\frac{k_{rj}}{\mu_j}\tilde{k}(\nabla P_j + \rho_j g\nabla h) \qquad \boxed{j = \text{phase}}$$

$$\text{1D:} \quad u_j = -\frac{kk_{rj}}{\mu_j}\left(\frac{\partial P_j}{\partial x} + \rho_j g\sin\alpha\right)$$

$$u_j = \frac{q_j}{A} \qquad u = \sum u_j$$

Mass Balance for Multiphase

if constant ϕ, ρ_j

$$\phi\frac{\partial S_j}{\partial t} + \frac{\partial u_j}{\partial x} = 0$$

$$\phi\frac{\partial S_j}{\partial t} + u\frac{\partial f_j}{\partial x} = 0$$

$$f_j = \frac{u_j}{u} \qquad \sum f_j = 1$$

Fractional Flow Derivation

1D flow of oil and water $\frac{u_o + u_w}{u} = 1$ Introduce capillary pressure $\frac{\partial P_c}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_w}{\partial x}$

Combine Darcy's Law, definition of fractional flow, and capillary pressure

$$f_w = \frac{1 + \frac{kk_{ro}}{u\mu_o}\left(\frac{\partial P_c}{\partial x} - \Delta\rho g\sin\alpha\right)}{1 + \frac{k_{ro}\mu_w}{k_{rw}\mu_o}} \xrightarrow[P_c = 0]{\text{if}} \frac{1 - N_g\sin\alpha}{1 + \frac{k_{ro}\mu_w}{k_{rw}\mu_o}} \xrightarrow[\alpha = 0]{\text{if}} \frac{1}{1 + \frac{k_{ro}/\mu_o}{k_{rw}/\mu_w}}$$

$$N_g = \frac{kk_{ro}\Delta\rho g}{u\mu_o} \qquad \alpha = \text{rsvr dip angle}$$

Fractional Flow – 1D Water Flood

Mobility Ratio

$$M = \frac{\lambda_{rw}}{\lambda_{ro}} = \frac{k_{rw}/\mu_w}{k_{ro}/\mu_o} \qquad \lambda_{rj} = \text{relative mobility}$$

Corey Equations

$$k_{rw} = k^o_{rw}S^n \qquad k_{ro} = k^o_{ro}(1 - S)^m$$

$$\boxed{S = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{or}}}$$

Fractional Flow

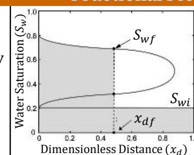
$$\boxed{f_w = \frac{1 - N_g^o(1 - S)^m\sin\alpha}{1 + \frac{(1 - S)^m}{M_o S^n}} \xrightarrow[\alpha = 0]{\text{if}} \frac{1}{1 + \frac{(1 - S)^m}{M_o S^n}}}$$

$$N_g^o = \frac{kk^o_{ro}\Delta\rho g}{u\mu_o} \qquad M_o = \frac{k^o_{rw}/\mu_w}{k^o_{ro}/\mu_o}$$

Important Definitions

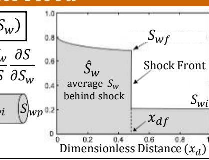
$$x_d = \frac{x}{L} \qquad t_d = \frac{ut}{\phi L}[=]\text{PVI} \quad \text{Pore Volumes Injected}$$

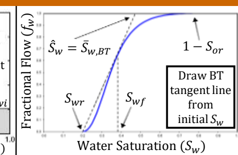
$$WOR = \frac{B_o}{B_w}\left[\frac{f_w|_{S_{wp}}}{1 - f_w|_{S_{wp}}}\right] \qquad N_P = \frac{V_p N_{pd}}{B_o}[=]\text{STB}$$



$$\boxed{x_d = t_d f'_w(S_w)}$$

$$f'_w = \frac{\partial f_w}{\partial S_w} = \frac{\partial f_w}{\partial S}\frac{\partial S}{\partial S_w}$$





Before Breakthrough

$$\bar{S}_w = S_{wf} + \frac{f_{wj} - f_w|_{S_{wf}}}{f'_w|_{S_{wf}}} \quad (f_{wj} \text{ usually } 1)$$

At Breakthrough

$$t_{d,BT} = \frac{1}{f'_w|_{S_{wf}}}$$

$$N_{Pd,BT} = t_{d,BT}f_{oi} \qquad f_{oi} = 1 - f_w|_{S_{wi}}$$

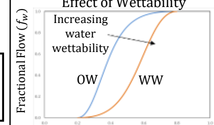
After Breakthrough

$$t_d = \frac{1}{f'_w|_{S_{wp}}} \qquad N_{Pd} = \bar{S}_w - S_{wi}$$

$$\bar{S}_w = S_{wp} + \frac{f_{wj} - f_w|_{S_{wp}}}{f'_w|_{S_{wp}}} \quad (f_{wj} \text{ usually } 1)$$

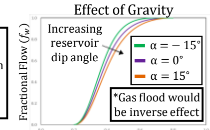
$\bar{S}_w$ = avg. S_w in swept area

Effect of Wettability

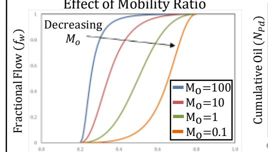


As f_w curve shifts right, breakthrough occurs later and produce oil quicker

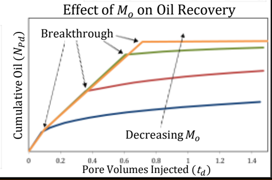
Effect of Gravity



Effect of Mobility Ratio



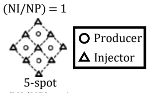
Effect of M_o on Oil Recovery



Well Patterns

Rules of Thumb

- For viscous oil (NI/NP) < 1
- For low viscosity oil or condensate (NI/NP) > 1
- For similar fluid viscosities (NI/NP) = 1



Buckley-Leverett Extension to 2-D

Definitions

$$Q_i^* = \frac{W_i}{V_p E_A} = \frac{1}{f'_w|_{S_{w2}}}$$

Q_i^* = effective t_d

$$\boxed{M_{\bar{S}} = \frac{(\lambda_{rw} + \lambda_{ro})|_{\bar{S}_{wf}}}{(\lambda_{rw} + \lambda_{ro})|_{S_{wi}}}}$$

$\bar{S}_{w2} = \bar{S}_{w5} = \bar{S}_w \qquad \bar{S}_{wf} = \bar{S}_w$

$S_{w2} = S_{w5} = S_{wp} = S_w|_{x_d=1}$

$W_i < W_{i,BT}$

$$E_{A,BT} = 0.546 + \frac{.0317}{M_{\bar{S}}} + \frac{0.30223}{e^{M_{\bar{S}}}} - .005097M_{\bar{S}}$$

$$W_{i,BT} = (V_p E_{A,BT})(\bar{S}_{wf} - S_{wi}) = N_{P,BT}$$

$$Q^*_{i,BT} = \frac{W_{i,BT}}{V_p E_A} = \bar{S}_{wf} - S_{wi}$$

$$\frac{\partial N_{Pu}}{\partial W_i} = \frac{1}{W_i}\left[\frac{0.274 W_{i,BT}(S_{wf} - S_{wi})}{E_{A,BT}(\bar{S}_{wf} - S_{wi})}\right]$$

$W_{i,BT} < W_i < W_{i,100}$

$$Q_i^* = Q^*_{i,BT}\{1 + a_1 e^{-a_1}[Ei(a_2) - Ei(a_1)]\}$$

$$a_1 = 3.65E_{A,BT} \qquad a_2 = a_1 + \ln(W_i/W_{i,BT})$$

$$\boxed{E_A = E_{A,BT} + 0.274\ln(W_i/W_{i,BT})}$$

$$N_P = (\bar{S}_{w2} - S_{wi})E_A V_p; \quad \bar{S}_{w2} = S_{w2} + Q_i^*(1 - f_{w2})$$

$$WOR = \frac{(f_w|_{S_{wp}})\left(1 - \frac{\partial N_{Pu}}{\partial W_i}\right)}{(1 - f_w|_{S_{wp}})\left(1 - \frac{\partial N_{Pu}}{\partial W_i}\right) + \left(\frac{\partial N_{Pu}}{\partial W_i}\right)}$$

$W_i > W_{i,100}$ $(E_A = 1)$

$$W_{i,100} = W_{i,BT}e^{\left(\frac{1 - E_{A,BT}}{0.274}\right)}$$

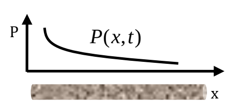
$$Q_i^* = Q^*_{i,100} + \frac{W_i - W_{i,100}}{V_p}$$

$$f_w|_{S_{w2}} = \left(\frac{WOR}{WOR + 1}\right)$$

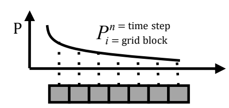
$$N_P = (\bar{S}_{w2} - S_{wi})V_p$$

Reservoir Simulation

Analytical (Continuous) Solution



Numerical (Discrete) Solution



Finite Differences Approximation

Taylor Series: $f(x + \Delta x) = f(x) + f'(x)\Delta x + \frac{1}{2!}f''(x)\Delta x^2 + \frac{1}{3!}f'''(x)\Delta x^3 + \dots$

Type	Deriv.	FD Approximation	Error
Forward	$\frac{\partial P}{\partial t}$	$\frac{P(t + \Delta t) - P(t)}{\Delta t}$	$O(\Delta t)$
Backward	$\frac{\partial P}{\partial t}$	$\frac{P(t) - P(t - \Delta t)}{\Delta t}$	$O(\Delta t)$
Centered	$\frac{\partial P}{\partial t}$	$\frac{P(t + \Delta t) - P(t - \Delta t)}{2\Delta t}$	$O(\Delta t^2)$
Centered	$\frac{\partial^2 P}{\partial x^2}$	$\frac{P(x + \Delta x) - 2P(x) + P(x - \Delta x)}{\Delta x^2}$	$O(\Delta x^2)$

Explicit: $\vec{P}^{n+1} = \vec{P}^n + \Delta t B^{-1}\left(\vec{Q} - T\vec{P}^n\right)$

Implicit: $\vec{P}^{n+1} = \left(T + \frac{1}{\Delta t}B\right)^{-1}\left(\frac{1}{\Delta t}B\vec{P}^n + \vec{Q}\right)$

Crank-Nicholson ($\theta = 0.5$)

$$\left\{(1 - \theta)T + \frac{1}{\Delta t}B\right\}\vec{P}^{n+1} = \left\{\frac{1}{\Delta t}B - \theta T\right\}\vec{P}^n + \vec{Q}$$

Boundary Condition

Dirichlet $(P(0, t) = P_{B1})$

0 | 1 $\quad P_B = \frac{P_0 + P_1}{2}; P_0 = 2P_B - P_1$

Neumann $(\partial P/\partial x = 0)$

N | N+1 $\quad \frac{P_N - P_{N+1}}{\Delta x} = 0; \; P_N = P_{N+1}$

Stability

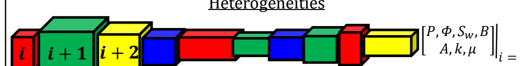
Explicit Method

$$\eta \equiv \frac{\alpha\Delta t}{(\Delta x)^2} \le 0.5 \; (1D)$$

CFL Condition

$$\frac{u\Delta t}{\Delta x} < 1$$ **Necessary for stability of IMPES

Heterogeneities



$$\left[\begin{matrix} P, \Phi, S_w, B \\ A, k, \mu \end{matrix}\right]_{i=1}$$

$k_r(S_{w,i})$

$k_r(S_{w,i+1})$

$$T_{i+\frac{1}{2}} = \left(\frac{k_r}{\mu B}\right)_{i+\frac{1}{2}}\left(\frac{kA}{\Delta x}\right)_{i+\frac{1}{2}} \qquad T = \frac{kA}{\mu B_w\Delta x}$$ Variables change from block to block

Upwinding

$$\left(\frac{k_r}{\mu B}\right)_{i+\frac{1}{2}} = \begin{cases} \frac{k_r(S_{w,i+1})}{\mu(P_{i+1})B(P_{i+1})} & \text{if } \Phi_{i+1} > \Phi_i \\ \frac{k_r(S_{w,i})}{\mu(P_i)B(P_i)} & \text{if } \Phi_i > \Phi_{i+1} \end{cases}$$

Harmonic Mean

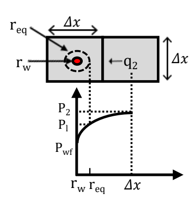
$$\left(\frac{kA}{\Delta x}\right)_{i+\frac{1}{2}} = \frac{2k_i A_i k_{i+1}A_{i+1}}{k_i A_i\Delta x_{i+1} + k_{i+1}A_{i+1}\Delta x_i}$$

Radial Solution

$$P_2 = P_l - \frac{q_w\mu B_w}{2\pi kh}\ln\left(\frac{\Delta x}{r_{eq}}\right) \qquad q_w = -J_l^w(P_l - P_{wf})$$

$$r_{eq} = \Delta x e^{-\frac{\pi}{2}} \approx 0.2\Delta x \qquad J_l^w = \frac{2\pi h\sqrt{k_x k_y}}{\mu B_w\{\ln(r_{eq}/r_w) + s\}}$$

$$r_{eq} = 0.28\frac{\left[(k_y/k_x)^{1/2}\Delta x^2 + (k_x/k_y)^{1/2}\Delta y^2\right]^{1/2}}{(k_y/k_x)^{1/4} + (k_x/k_y)^{1/4}}$$



IMPES (Implicit Pressure and Explicit Saturation)

Pressure (solve implicitly) | Saturation (solve explicitly)

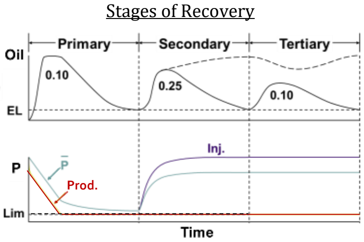
$$\left(\mathbf{T} + \mathbf{J} + \frac{\mathbf{B}}{\Delta t}\right)\mathbf{P}^{n+1} = \frac{\mathbf{B}}{\Delta t}\mathbf{P}^n + \mathbf{Q} \qquad \mathbf{S}_w^{n+1} = \mathbf{S}_w^n + \mathbf{d}_{12}[-\mathbf{T}_w\mathbf{P}^{n+1} + \mathbf{Q}_w] - \mathbf{C}_{tw}(\mathbf{P}^{n+1} - \mathbf{P}^n)$$

$$\mathbf{T} = \mathbf{T}_w + \frac{B_o}{B_w}\mathbf{T}_o \qquad \mathbf{J} = \mathbf{J}_w + \frac{B_o}{B_w}\mathbf{J}_o \qquad \mathbf{Q} = \mathbf{Q}_w + \frac{B_o}{B_w}\mathbf{Q}_o \qquad \mathbf{B}_i = \frac{V_i\phi_i c_{t,i}}{B_w}$$

$$\mathbf{C}_{tw,i} = S^n_{w,i}(c_{w,i} + c_{r,i}) \qquad c_t = c_w S_w + c_o S_o + c_R \qquad \mathbf{d}_{12,i} = \frac{V_i\phi_i}{B_{w,i}\Delta t}$$

Improved Oil Recovery

Stages of Recovery



**Oil Saturation constant during Primary Recovery

Primary Recovery

Natural Flow | Artificial Lift | Hydraulic Fracturing

Secondary Recovery

Waterflooding | Pressure Maintenance

Tertiary Recovery (EOR)

- Thermal
  - Steam
  - Hot Water
  - Combustion
- Gas Injection
  - Carbon Dioxide
  - Hydrocarbon
  - Nitrogen Flue
- Chemical
  - Alkali
  - Surfactant
  - Polymer
- Other
  - Microbial
  - Acoustic

IOR

IOR: Improved Oil Recovery

Adapted from SPE-SPE-8490B, 87864

3D Cumulative Oil Recovery

$$N_{pd} = E_A E_D E_I(S_{oi} - S_{or})$$

E_A = Areal Sweep Efficiency E_D = Displacement Efficiency

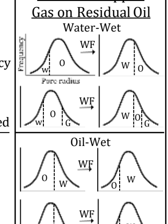
E_I = Vertical Sweep Efficiency

$$E_A = \frac{\text{Area contacted by fluid}}{\text{Total area}} \qquad E_D = \frac{\text{Amount of oil displaced}}{\text{Total moveable oil}}$$

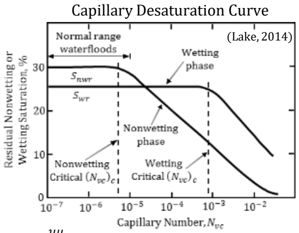
$$E_I = \frac{\text{Cross−sectional area contacted by fluid}}{\text{Total cross−sectional area}}$$

E_A is a $f(M_{\bar{S}}, t_d, \text{pattern})$ $\qquad E_D = \frac{S_{oi} - \bar{S}_o}{1 - S_{wr} - S_{or}}$

Effect of Trapped Gas on Residual Oil



Capillary Desaturation Curve



$$N_{vc} = \frac{u\mu}{\sigma}[=](viscous\ forces/capillary\ forces)$$

Created by James Riddle with guidance from Dr. Matthew T. Balhoff and contributions from Jenny Ryu and Matt Mlynski

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