The University of Texas at Austin
Hildebrand Department of Petroleum and Geosystems Engineering
Cockrell School of Engineering

Fundamentals of Drilling Engineering

Conversions and Constants

1 kPa = 0.1450 psi	1 in = 2.54 cm	1 acre = 43,560 ft^2	1 m^3 = 6.2898 bbl	R = 459.67 + °F	1 lb$_m$ = 453.59 g
1 MPa = 10 bar	1 ft = 0.3048 m	1 m^2 = 10.764 ft^2	1 bbl = 5.6146 ft^3	K = 273.15 + °C	
1 atm = 14.696 psi	1 mile = 5,280 ft		1 bbl = 42 US gal	°F = 1.8 °C + 32	
1 atm = 1.013 bar			1 ft^3 = 7.4805 gal		1 cp = 1.0 mPa-s

1 bar = 1 x 10^6 dynes/cm^2	1 lb$_m$/gal = 0.052 psi/ft	Standard Pressure = 14.696 psia
1 Newton = 1 x 10^5 dynes	1 mD/cp = 6.33 x 10^{-3} ft^2/psi–day	Standard Temperature = 60°F
1 dyne = 2.248 x 10^{-6} lb$_f$	1 Darcy = 9.8692 x 10^{-9} cm^2	Gas Constant = 10.732 psia · ft^3/lbmol · R
1 g/cm^3 = 62.428 lb$_m$/ft^3	1 hp = 0.7457 kw	Water density at SC = 62.37 lb$_m$/ft^3
	1 hp = 33,000 ft · lb$_f$/min	Molar Mass of Air = 28.966 g/mol
1 kg/l = 8.347 lb$_m$/gal	1 BTU = 778 ft · lb$_f$	V_M^{IG} = 379.3 scf/lbmol @ 14.696 psia

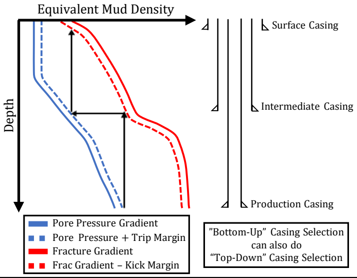
Casing Design

Recommended Clearance Ratio

Clearance between tube centered in larger tube: $0.13 \le \frac{C}{OD_{in}} \le 0.18$ $C = \frac{ID_{out} - OD_{in}}{2}$

OD_{in} = inside tube OD ID_{out} = outer tube ID

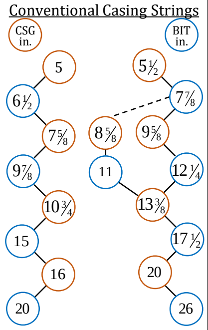
Mud Window and Casing Point Selection



Common Bit Sizes

Casing Size (OD in.)	Common Bit Sizes (in.)
4½	6, 6⅛, 6¼
5	6½, 6¾
5½	7⅞, 8⅜
6	7⅞, 8⅜, 8½
6⅝	8½, 8⅝, 8¾
7	8⅝, 8¾, 9½
7⅝	9⅞, 10⅝, 11
8⅝	11, 12¼
9⅝	12¼, 14¾
10¾	15
13⅜	17½
16	20
20	24, 26

Conventional Casing Strings



Rig Power Requirements

Hoisting Power

$P_{input} = \frac{W v_b}{E(33{,}000)}[=]hp$

W = hook load$[=]lb_f$
v_b = velocity of blocks$[=]ft/min$
E = hoisting efficiency
n = number of lines strung through blocks

Hoisting Efficiency

n	E
6	0.874
8	0.841
10	0.810
12	0.770
14	0.740

Rotating Power

$P_T = \frac{wT}{33{,}000}[=]hp$

$w = 2\pi N[=]rad/min$
N = rotary speed$[=]rpm$
T = rotary torque$[=]ft \cdot lb_f$
P_T = torque horsepower

Pumping Power

$H_{input} = \frac{q P_d}{1714\eta}[=]hp$

q = flow rate$[=]gal/min$
P_d = discharge pressure$[=]psi$
η = overall pump efficiency, normally $\eta = 0.9$

Pumps

Single Acting Pumps

$q = 0.0034(d_p^2 N_p L_s N \eta_v)[=]gal/min$

d_p = plunger diameter$[=]in$ N_p = number of plungers
N = pump speed$[=]strokes/min$ L_s = stroke length$[=]in$
η_v = volumetric efficiency which is usually between 0.8 – 0.9

Down Hole Motors

Output Shaft Torque

$T = 3.064\frac{q\eta_v \Delta P}{N}[=]ft \cdot lb_f$

$\Delta P = \Delta P$ through motor | $N[=]rpm$ | η_v = volumetric efficiency

Key Notes on Motors

- Motor provides torque at the bit depending on q and ΔP
- A motor is a speed multiplier, adding to top drive or rotary speed
- Torque from the motor helps the bit drill in highly deviated sections where it's difficult to apply WOB
- q, ΔP, and N are interrelated and depend on how motor is made
- If motor cannot supply the required torque to break rock, the motor will stall and motor damage can occur

Drilling Fluid Considerations

Fluid Pressure Gradient

$\nabla P = \rho/144\,[=]\,psi/ft$

$\rho[=]lb_m/ft^3$

$\nabla P/0.052\,[=]\,lb_m/gal$

Rotational Viscometer

$\mu = \theta_{600} - \theta_{300}[=]cp$

$\tau_y = \theta_{300} - \mu[=]\frac{lb_f}{100\,ft^2}$

Viscosity is Newtonian if yield point is zero, if not it's plastic viscosity

Plastic Viscosity Limits

$\mu_{max} = 2.94\exp(0.164\rho_m)$

$\mu_{min} = 1.58\exp(0.171\rho_m)$

$\mu\,[=]cp$

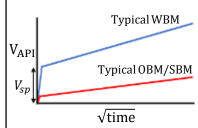
Filtration Rate – API Tests

$V_{30} = 2(V_{7.5} - V_{sp}) + V_{sp}$

V_{30} = API vol. collected at 30 mins
$V_{7.5}$ = volume collected at 7.5 mins
V_{sp} = spurt vol. collected at 0 mins

$V_{sp} = V_1 - \frac{V_2 - V_1}{\sqrt{t_2} - \sqrt{t_1}}\sqrt{t_1}$

When conducting HPHT test, V_{sp} & V_{30} must be multiplied by 4 due to smaller HPHT filter cell



Adjusting Weight with Barite

$m_{Ba} = 42(V_f - V_i)\rho_{Ba}[=]lb_m$

$V_f = V_i\left(\frac{\rho_{Ba} - \rho_i}{\rho_{Ba} - \rho_f}\right)[=]bbl$ $\rho_{Ba} = 35ppg$

Mud Quality Control

$f_{lg} = \frac{f_s(\rho_{Ba} - \rho_w) - \rho_m + \rho_w - f_o(\rho_w - \rho_o)}{\rho_{Ba} - \rho_{lg}}$

$f_{Ba} = f_s - f_{lg}$ $\rho_{lg} = 21.7ppg$

WBM Maximum Solids Fraction

$f_{s,max} = 0.0289\rho_m - 0.139$

In general, the OBM $f_{s,max}$ is about 30%

Limits for Yield Point

$\tau_{y,max} = 353\rho_m^{1.15}$ | $\tau_{y,min} = 0.07\rho_m - 0.45$

Altering suspended low γ solids normally keeps mud parameters within recommended ranges

Routine Pipe Calculations

Capacity and Volume

$C_{pipe} = \frac{ID^2}{1029.4}$ $C_{annular} = \frac{OD^2 - ID^2}{1029.4}$

$C[=]bbl/ft$ | $V[=]bbl$ | $ID, OD[=]in$ | $L[=]ft$

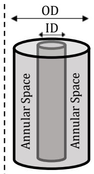
$V_{pipe} = \frac{ID^2 L}{1029.4}$ $V_{annular} = \frac{(OD^2 - ID^2)L}{1029.4}$

Using Pipe Weight

$C_{pipe} = \frac{OD^2}{1029.4} - \frac{W_{pipe}}{5.615\rho_{pipe}}$

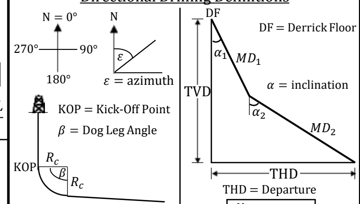
$W_{pipe}[=]\frac{lb_f}{ft}$ $\rho_{pipe}[=]\frac{lb_m}{ft^3}$

$ID = 24\sqrt{\left(\frac{OD}{24}\right)^2 - \frac{W_{pipe}}{\pi\rho_{pipe}}}$



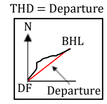
Well Trajectory

Directional Drilling Definitions



$DLS = \frac{5729.6}{R_c}[=]\frac{\text{deg.}}{100\,ft}$

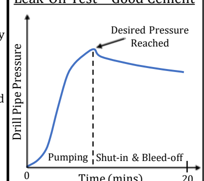
DLS = Dog Leg Severity



$\beta = 2\arcsin\sqrt{\sin^2\left(\frac{\alpha_2 - \alpha_1}{2}\right) + \sin\alpha_1\sin\alpha_2\sin^2\left(\frac{\varepsilon_2 - \varepsilon_1}{2}\right)}$

Cement

Leak-Off Test – Good Cement



Common Properties

API Class	A	H
Water (%) by Cement Weight	46	38
Water (gal/sack)	5.19	4.29
Slurry Density (ppg)	15.6	16.4
Slurry Yield (ft^3/sack)	1.18	1.06

24 Hour Compressive Strength (psi)

Curing Temperature and Pressure (psi)

Class A Cement	60°F 0	80°F 0	95°F 800	110°F 1600	140°F 3000
	3000	4050	5500	6700	8400
Class H Cement	95°F 800	110°F 1600	140°F 3000	170°F 3000	200°F 3000
	615	1905	2085	2925	5050

Drill String Considerations

Weight Indicator

$W_{ind} = W_{B,DS} + F_D + W_{TE}[=]lb_f$

F_D (−) value moving down F_D (+) value moving up

Required Drill Collar Length

$L_{DC} = \frac{SF \cdot WOB}{W_{B,DC}\cos\alpha}[=]ft$

$W_{B,DC}$ = buoyed drill collar WOB = weight on bit

Linear Pipe Drag

$F_D = \mu N = \mu F\sin\alpha$ $F = F_{ax} - W_B$

N = normal force F_{ax} = axial force

μ = friction coefficient and is usually 0.15 – 0.6

Rotational Pipe Drag

$T = \mu\left(\frac{OD}{2}\right)F\sin\alpha[=]in \cdot lb_f$

When rotating pipe up or down while circulating mud, the overall drag is normally reduced and not additive

Buoyed Pipe Weight

$W_B = W - W_f[=]lb_f/ft$ $W_B = W(1 - \rho_m/\rho_{pipe})$

W = pipe air weight W_f = fluid weight $W_B = W(1 - 0.0153\rho_m)$

W_B = buoyed pipe weight $\rho_m[=]ppg$ $\rho_{steel} = 65.5\,ppg$

Buckling – Vertical Hole

$W_{cr,1}$ = 1st order buckle $= 1.94mW_{B,DC}\,[=]lb_f$

$W_{cr,2}$ = 2nd order buckle $= 3.75mW_{B,DC}[=]lb_f$

$m = \sqrt[3]{\frac{EI}{W_{B,DC}}}$

$I = \frac{\pi}{64}(OD^4 - ID^4)$

Critical WOB for deviated hole usually outside recommended range for effective drilling

Maximum Pull on Pipe

$F_{max} = \frac{\sigma_y}{SF}A[=]lb_f$

σ_y = pipe min. yield strength$[=]psi$

$A = \frac{\pi}{4}(OD^2 - ID^2)[=]in^2$

Depth to Stuck Point

$L = E\frac{\Delta l}{\Delta F}A[=]ft$

Δl = drill pipe stretch$[=]ft$

ΔF = incremental force pulled against stuck drill pipe$[=]lb_f$

Von Mises Effective Stress Criterion

$\sigma_e < 0.8\sigma_y$

$$\sigma_e = \sqrt{\left(\frac{F_{ax} + \Delta P\frac{\pi}{4}ID^2}{\frac{\pi}{4}(OD^2 - ID^2)}\right)^2 - \frac{\Delta P(OD)}{2t}\left(\frac{F_{ax} + \Delta P\frac{\pi}{4}ID^2}{\frac{\pi}{4}(OD^2 - ID^2)}\right) + \left(\frac{\Delta P(OD)}{2t}\right)^2 + 3\left(\frac{T(OD)}{2J}\right)^2}$$

ΔP = pump pressure to overcome pipe friction and bit nozzle losses$[=]psi$ $J = 2I[=]in^4$

Casing Loads

Collapse Pressure

$P_c = SF(P_e - P_i) + P_T$

$SF = 1.125$

Collapse resistance determined by four different eqns based on the OD/t ratio

(more info: API Bulletin 5C3)

Burst Pressure

$P_b = SF(P_i - P_e) + P_T$

P_{BR} = burst resistance

$P_{BR} = DF\left(\frac{2\sigma_y t}{OD}\right)$

$DF = 0.875$ $OD, t[=]in$

Worst Case Scenario for Collapse and Burst

Collapse: Lost Circulation During Drilling

$P_c = SF(0.052\rho_m)D$ D = csg shoe depth$[=]ft$

Burst: Gas Kick Fills Casing

Pressure high enough to fracture rock at shoe of the casing

$P_{iS} = P_{iD} - P_g$ P_{iS} = P inside casing at the surface

$P_{iD} = P_{frac} + \Delta P_{fric}$ P_{iD} = pressure inside casing at D

$\Delta P_{fric} \approx 0.015D$ P_{frac} = fracture pressure at D

$P_g = 0.0187D\left[\frac{P_{iD}\gamma_g}{T_D}\right]$ P_g = estimated (CH_4) gas column P

T_D = temperature at $D[=]R$

$T_D = D\alpha_T + T_{surface}$ α_T = local temperature gradient

Casing in Tension

$F_t = SF(W)D_b + F_T + F_B[=]lb_f$ $SF = 1.8$

$F_B = 64(DLS)(OD)W\frac{6KL_j}{\tanh(6KL_j)}$ $L_j[=]ft$ $K[=]in^{-1}$

$K = \sqrt{WD_b/(EI)}$

Well Control

Pressure Balance

$P_{BH} = P_{dp} + 0.052\rho_m TVD$ $\rho_m[=]\frac{lb_m}{gal}$

$P_{BH} = P_{csg} + 0.052\rho_m TVD$

Post-Kick Calculations

$h_{kick} = \frac{C_{annular}}{V_{pit}}[=]ft$ $C_{ann}[=]\frac{bbl}{ft}$

V_{pit} = mud pit gain$[=]bbl$

$h'_{kick} = h_{kick}\cos\alpha$

h'_{kick} = true vertical height of kick

$\rho_{kick} = \rho_m + \frac{(P_{dp} - P_{csg})}{0.052h'_{kick}}[=]ppg$

$P_{rsvr} = P_{dp} + 0.052\rho_m TVD$

$\rho_{kill} \ge \frac{P_{rsvr}}{0.052TVD}[=]ppg$

$m_{Ba} = \frac{V_m\rho_{Ba}(\rho_{kill} - \rho_m)}{\rho_{Ba} - \rho_{kill}}[=]lb_m$

V_m = total system mud volume$[=]gal$

$V_{inc} = \frac{V_m(\rho_{kill} - \rho_m)}{42(\rho_{Ba} - \rho_{kill})}[=]bbl$

Post-Kick Important Actions

- Record shut-in drill pipe pressure
- Record shut-in casing pressure
- Record mud pit volume gain
- If shut-in pressures increase over time, reservoir fluid is gas
  - If not, kick is liquid

Kick Indicators

- Flow rate/pit volume increase
- Pump pressure decrease along with pump stroke increase
- Improper hole fill-up on trips
- Change in drill string weight
- Unexpected drilling rate increase

Kick Control Methods

Driller's Method
- Two circulations
  1. Circulate out kick
  2. Kill the well

Weight & Wait
- One circulation
  1. Circulate out kick and kill well

Circulating Pressures

$P_{ic} = P_{dp} + P_{csr}$

$P_{fc} = P_{csr}(\rho_{kill}/\rho_m)$

P_{csr} = circulating pressure at slow pump rate

Equivalent Circulating Density

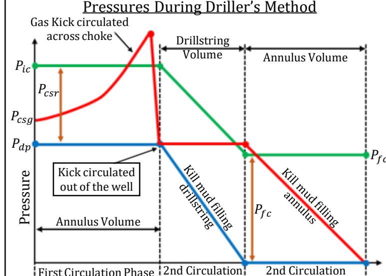
$ECD = \frac{\nabla P \cdot TVD + \Delta P_{Ann}}{0.052TVD}[=]ppg$

Effective mud weight during dynamic circulating conditions

Number of Pump Strokes

$STK_{dp} = \frac{V_{dp} + V_{dc}}{42q_p}$ $V[=]bbl$ $q_p[=]gal/stroke$ $STK_{ann} = \frac{V_{dp,ann} + V_{dc,ann}}{42q_p}$

Pressures During Driller's Method



Hydraulics

Pump Pressure

$P_p = \frac{\rho}{6704}\frac{q^2}{d_e^4} + \sum\frac{\bar{v}^2}{25.8}\frac{L}{D}f\rho\,[=]psi$

d_e = equivalent nozzle diameter $= \left(\sum_{i=1}^{n} d_i^2\right)^{1/2}$

d_i = bit nozzle diameter$[=]in$

$\rho[=]ppg$ | $q[=]gal/min$ | $\bar{v}[=]ft/sec$ | $L[=]ft$ | $D[=]in$

Optimizing Bit Hydraulics

- Normally it's only possible to optimize bit hydraulics if drilling without a motor
- Adjust bit nozzle diameters to consume 64% of total P_p across the bit to maximize hydraulic power
- Adjust bit nozzles to consume 46% of total pump pressure across bit to maximize jet impact force

$HP_{bit} = \frac{\Delta P_{bit}q}{1714}$ $HSI = \frac{1.273HP_{bit}}{d_e^2}[=]\frac{hp}{in^2}$

$q[=]gal/min$ HP_{bit} = bit hydraulic horsepower

Viscosity

If fluid is a Bingham Plastic, calculate apparent viscosity

$\mu_a = \mu_p + 6.66\frac{\tau_y D}{\bar{v}}$

Reynold's Number

$N_{Re} = 928\frac{D v \rho}{\mu}$

*units same as Pump Pressure

$N_{Re} < 2100$: Laminar Flow

$N_{Re} > 2100$: Turbulent Flow

Friction Factor

$f = aN_{Re}^{-b}$

Laminar	Turbulent
$a = 16$	$a = 0.0791$
$b = 1$	$b = 0.25$

Hole Cleaning

This eqn estimates the added mass of rock

$m_r = (ROP)A_b\rho_{rock}(1 - \phi)$

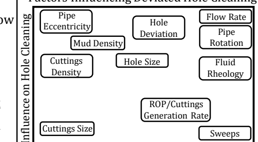
m_r [=]rock mass per time

ROP = rate of penetration$[=]ft/hr$

A_b = bit area$[=]ft^2$ $\rho_{rock}[=]lb_m/ft^3$

If cuttings from shaker cleaned and weighed as a function of time, a rough idea of the state of cleaning can be obtained

Factors Influencing Deviated Hole Cleaning



Active Control in Field Practice (SPE 132372, 2010)

Nomenclature

Common Variables

OD, ID = outer, inner diameter
ρ_m = mud density$[=]ppg$
q = flow rate$[=]gal/min$
σ_y = min. yield strength$[=]psi$
E = Young's modulus
$E = 30x10^6 psi$ for steel
I = moment of inertia
J = polar moment of inertia
SF = safety factor
DF = design factor
t = pipe wall thickness$[=]in$

Drilling Fluid Considerations

$\theta_\#$ = reading at # rpm
τ_y = yield point
m_{Ba} = Barite mass to add
ρ_{Ba} = Barite density
ρ_i = initial mud density$[=]ppg$
ρ_f = final mud density$[=]ppg$
f_{lg} = low gravity solids fraction
ρ_w = base water $[=]ppg$
ρ_o = oil density$[=]ppg$
f_s = fraction of total solids
f_o = oil fraction
ρ_{lg} = drilled low gravity solids
$\tau_{y,max}, \tau_{y,min}[=]lb_f/100\,ft^2$

Rig Power Requirements

w = angular velocity

Well Trajectory

R_c = radius of curvature
TVD = true vertical depth
MD = measured depth
THD = total horizontal distance
BHL = bottom hole location

Routine Pipe Calculations

W_{pipe} = pipe air weight

Pumps

q = flow rate under load (ΔP)

Down Hole Motors

N = shaft speed

Drill String Considerations

F_D = drag due to friction
W_{TE} = traveling equipment
$W_{B,DS}$ = buoyed drill string
α = hole inclination$[=]deg$
T = rotating torque
W_{cr} = critical weight on bit

Casing Loads

P_e = external pressure
P_i = internal pressure
P_T = P due to temp change
F_t = tensile load
W = casing air weight
D_b = depth below bend
F_T = force due to temp change
F_B = force due to bending
DLS = dogleg severity
K = bending constant
L_j = joint length
ΔP_{fric} = P to overcome friction
γ_g = gas specific gravity

Well Control

P_{BH} = P at depth TVD$[=]psi$
P_{dp} = shut–in drill pipe P
P_{csg} = shut–in casing P
α = average inclination angle
TVD = true vertical depth
m_{Ba} = Barite mass to add
P_{rsvr} = formation $P[=]psi$
ρ_{kill} = kill mud density
V_{inc} = volume increase
P_{ic} = initial circulating P
P_{fc} = final circulating pressure
∇P = mud pressure gradient
ΔP_{Ann} = frictional P loss in annulus above point of interest
V_{dp} = drill pipe volume
V_{dc} = drill collar volume
q_p = pump output
STK_{dp} = strokes to fill drill pipe
$V_{dc,ann}$ = drill collar annular V
$V_{dc,ann}$ = drill collar annular V
STK_{ann} = strokes to fill annulus

Hydraulics

$\bar{v}$ = avg. fluid velocity in pipe
f = friction factor
L = pipe segment length
D = flow conduit diameter
μ_a = apparent viscosity$[=]cp$
ρ_{rock} = rock grain density

Created by James Riddle with guidance from Dr. Paul Bommer and Dr. Matthew T. Balhoff
Contact balhoff@mail.utexas.edu with comments/suggestions

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Fundamentals of Production Engineering

Nomenclature

Corrosion

$\Delta mass$ = mass lost
Δt = test duration
ρ_c = coupon density
A_s = coupon surface area

Flow in Pipes

f = friction factor
ID = internal diameter[=]in
L = pipeline length[=]ft
γ_g = gas specific gravity
MW_{air} = molar mass of air
μ = fluid viscosity[=]cp
N_{Re} = Reynold's number
P_1 = inlet pressure[=]psi
P_2 = outlet pressure[=]psi
q = liquid rate[=] bbl/day
q_{sc} = gas rate[=] $Mscf/day$
ρ = fluid density[=]lb_m/ft^3
$\bar{T}$ = avg. flowing temp[=]R
$\bar{v}$ = avg. velocity[=]ft/sec
$\bar{z}$ = avg. z-factor

Flow Control

D_c = choke diameter[=]in
ID_p = upstream diameter[=]in
ΔP = ΔP across choke[=]$psia$
$\hat{C}_p$ = constant pressure specific heat capacity[=] $BTU/lb_m°F$
$D_{64} = 64(D_c)$[=]$in/64$
T_1 = upstream temp[=]R

Pipeline Design

σ_y = min. yield strength[=]psi
t = wall thickness[=]in
OD = outer diameter[=]in
P_i = internal pressure[=]psi
P_{MT} = mill test pressure[=]psi
P_b = burst pressure[=]psi
P_d = design pressure[=]psi

IPR

q_o^{sc} = oil rate[=]STB/day
P_e = boundary pressure[=]psi
P_{wf} = flowing wellbore[=]psi
k_o = oil permeability[=]mD
h = reservoir thickness[=]ft
B_o = oil FVF[=]RB/STB
μ_o = oil viscosity[=]cp
r_e = drainage radius[=]ft
r_w = wellbore radius[=]ft
s = skin factor
$\bar{P}$ = avg. reservoir P[=]psi
k_g = gas permeability[=]mD
q_g^{sc} = gas rate[=] $Mscf/day$
$\bar{\mu}_g$ = avg. gas viscosity[=]cp
T = reservoir temp[=]R

Predicting Gas Production

G_P = cumulative gas produced
G_r = recoverable gas
G = original gas in place

Decline Curve Analysis

b = hyperbolic exponent
q = future rate [=] $prod/time$
q_i = initial rate[=] $prod/time$
t = time
D = decline rate[=] $1/time$
D_i = initial decline[=] $1/time$
N_p = cumulative production

Artificial Lift

PIP = pump intake P
PDP = pump discharge P
ΔP_{SV} = ΔP thru standing valve
$P_{s,tbg}$ = tbg surface pressure
∇P_{tbg} = tbg fluid P gradient
S = surface stroke length[=]in
s_{tbg} = tubing stretch[=]in
s_{rod} = rod stretch[=]in
$MPRL$ = min. polish road load
$PPRL$ = peak polish rod load
F_o = fluid weight[=]lb_f
W_D = dynamic load[=]lb_f
L_p = plunger seal length[=]in

Packer Forces

P_a = P above the packer
P_b = P below the packer
ID_s = seals ID[=]in
A_{pkr} = cross sectional area
ΔP_{ann} = avg. ΔP in annulus
ΔP_{tbg} = avg. ΔP in the tubing
δ = linear thermal expansion

Resource Economics

NRI = net revenue interest
OP = oil price
AVT = ad valorem tax
GP = gas price
GOR = gas–oil ratio
NP_g = net gas price
DCF = discounted cash flow

Flow in Pipes

Mechanical Energy Balance

Liquid Flow

Consistent Units

$$\frac{8q^2}{\pi^2}\left(\frac{1}{ID_2^4}-\frac{1}{ID_1^4}\right) + gL\sin\theta + \frac{g}{\rho}(P_2-P_1) + 2\bar{v}^2\frac{L}{ID}f = 0$$

Kinetic energy | Potential energy, vertical flow $\theta = 90°$ | Expansion or Compression | Energy of friction

Gas Flow

Consistent Units

$$\int_{P_1}^{P_2}\frac{zRT}{\gamma_g MW_{air}P}dP + gL\sin\theta + \frac{32fL}{\pi^2 ID^5}\left(\frac{q_{sc}zTP_{sc}}{PT_{sc}}\right)^2 = 0$$

horizontal $\theta = 0°$

Flow in pipes ΔKE negligible

Horizontal Gas Flow

$$P_2^2 - P_1^2 = 1.0068x10^{-4}\left[\frac{\gamma_g\bar{z}\bar{T}q_{sc}^2}{ID^5}\{fL + ID\ln(P_1/P_2)\}\right]$$

Inclined Gas Flow

$$P_2^2 = e^sP_1^2 + 2.685x10^{-3}\frac{f(\bar{z}\bar{T}q_{sc})}{ID^5\sin\theta}(e^s-1) \qquad s = \frac{-0.0375\gamma_g L\sin\theta}{\bar{z}\bar{T}}$$

Friction Factor

Moody correlations based on Reynolds Number

$$N_{Re} = 1.478\frac{q\rho}{ID\mu}$$

$$N_{Re} = 20.09\frac{\gamma_g q_{sc}}{ID\mu}$$

Laminar Flow

$$N_{Re} \le 2100$$

$$f = \frac{16}{N_{Re}}$$

Turbulent Flow (in a smooth tube)

$$N_{Re} > 2100$$

$$f = \frac{0.0791}{N_{Re}^{0.25}}$$

Flow Control at the Surface

Liquid Flow – Choke

Isothermal, steady-state mechanical energy balance

$$\frac{1}{2}(v_2^2 - v_1^2) = g\frac{P_1 - P_2}{\rho}$$

$$q = \frac{8081.7C(D_c^2)}{\sqrt{1-\left(\frac{D_c}{ID_p}\right)^4}}\sqrt{\frac{\Delta P}{\rho}}$$

Discharge Coefficient (C)

$$C = \frac{\text{actual flow rate}}{\text{theoretical rate}}$$

Empirical Estimate

$$C = C_1 - 6.53\sqrt{\frac{D_c}{ID_p N_{Re}}}$$

$C_1 = 0.9975$

Gas Flow Through a Choke

specific heat ratio $\gamma_H \approx \frac{\hat{C}_p MW_{air}\gamma_g}{\hat{C}_p MW_{air}\gamma_g - 1.99}$ → critical value $CV = \left(\frac{2}{\gamma_H + 1}\right)^{\frac{\gamma_H}{\gamma_H - 1}}$

$\left(\frac{P_2}{P_1}\right)_m \le CV$ — Critical flow achieved

$\left(\frac{P_2}{P_1}\right)_m$ uses measured pressures from well

P_1 = upstream pressure
P_2 = downstream pressure

$\left(\frac{P_2}{P_1}\right)_m > CV$ — Critical flow NOT achieved

$$q = 0.238CD_{64}^2P_1\sqrt{\frac{\gamma_H\left\{CV^{\left(\frac{2}{\gamma_H}\right)} - CV^{\left(\frac{\gamma_H+1}{\gamma_H}\right)}\right\}}{\gamma_g T_1(\gamma_H - 1)}}$$

Critical Flow Not Achieved

$$q = 0.238CD_{64}^2P_1\sqrt{\frac{\gamma_H}{\gamma_g T_1(\gamma_H - 1)}\left[\left(\frac{P_2}{P_1}\right)_m^{\frac{2}{\gamma_H}} - \left(\frac{P_2}{P_1}\right)_m^{\frac{\gamma_H+1}{\gamma_H}}\right]}$$

Inflow Performance Relation (IPR)

Oil Productivity Index

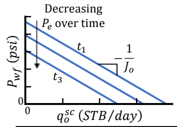
$$J_o = \frac{q_o^{sc}}{P_e - P_{wf}}[=]\frac{STB}{psi \cdot day}$$

Steady-state & radial flow

$$J_o = \frac{0.00708k_o h}{B_o\mu_o\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2} + s\right]}$$

Only good for single phase flow

Single-Phase Oil IPR



If B_o, μ_o, k_o relatively constant, J_o is a constant

Gas Productivity Index

$$J_g = \frac{q_g^{sc}}{P_e^2 - P_{wf}^2}[=]\frac{Mscf}{psi^2 \cdot day}$$

Apply steady-state & radial flow

$$J_g = \frac{7.0225x10^{-4}(k_g h)}{\bar{\mu}_g\bar{z}T\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2} + s\right]}$$

Gas properties evaluated at $\bar{P}$

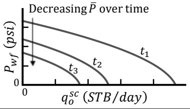
Two-Phase IPR ($P < P_{BP}$)

Empirical correlation (Vogel, 1968)

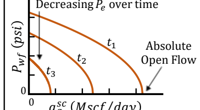
$$\frac{q_o}{q_{o,max}} = 1 - 0.2X - 0.8X^2$$

$$X = P_{wf}/\bar{P}$$

Can replace $\bar{P}$ with P_e



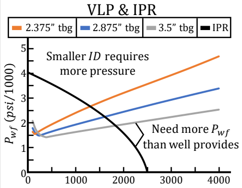
Gas IPR Curves



Vertical Lift Performance

- Vertical lift performance can be developed by using the mechanical energy balance
- VLP displays bottom hole pressure required to flow to surface at varying flow rates

VLP & IPR



IPR < VLP need artificial lift to flow to surface

Decline Curve Analysis

Hyperbolic Decline

(Arps, 1940)

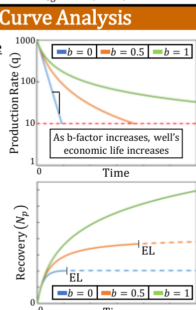
$$0 < b < 1$$

$$q = \frac{q_i}{(1 + bD_i t)^{1/b}}$$

$$t = \frac{1}{D_i b}\left\{\left(\frac{q_i}{q}\right)^b - 1\right\}$$

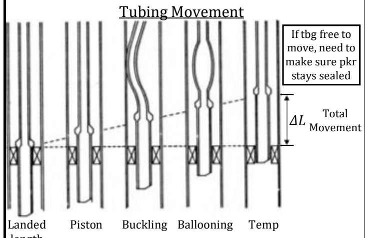
$$N_p = \frac{(q_i - q_i^b q^{1-b})}{D_i(1-b)}$$

$$D = D_i\left(\frac{q}{q_i}\right)^b$$



Packer Forces

Tubing Movement



$$\Sigma F = F_b + F_a + F_{bu} + F_{ba} + F_T \qquad \Delta L = \frac{\Sigma F}{A_{tbg}}\frac{MD}{E}[=]ft$$

F_a = force acting on seals from above MD = pkr measured depth
F_b = force acting on seals from below A_{tbg} = cross sectional area

- If tbg cannot move, need to check tensile strength of pkr and tbg

$$F_{top} = MD(W_B) - F_T - F_{ba} - F_{SO}$$

F_{top} = force at top of tubing W_B = buoyed tbg weight[=]lb_f/ft

Temperature Change

(for steel) $E = 30 \times 10^6\ psi$, $\delta = 6.9 \times 10^{-6}\ °F^{-1}$

$$F_T = A_{tbg}(\Delta T)E\delta = 207A_{tbg}(\Delta T)$$

Piston Forces

$$F_a = F_{SO} + P_a\left[\frac{\pi}{4}\left(OD_b^2 - OD_s^2\right)\right]$$

$$P_a = P_{s,ann} + D_{pkr}\nabla P_{ann}$$

F_{SO} = slacked off tubing weight
OD_b = packer bore diameter[=]in
OD_s = metal seal tube OD[=]in
D_{pkr} = packer true vertical depth
$P_{s,ann}$ = surface pressure in annulus
∇P_{ann} = annulus fluid pressure gradient

$$F_b = -P_b\left[\frac{\pi}{4}\left(OD_b^2 - ID_s^2\right)\right]$$

$$P_b = P_{s,tbg} + D_{pkr}\nabla P_{tbg}$$

Permanent Buckling

$$F_{bu} = A_{pkr}(\Delta P_{ann} - \Delta P_{tbg})$$

$$\Delta P_{ann} = \Delta P_{s,ann} + \frac{D_{pkr}\nabla P_{ann}}{2}$$

$$\Delta P_{tbg} = \Delta P_{s,tbg} + \frac{D_{pkr}\nabla P_{tbg}}{2}$$

Tubing Ballooning

$$F_{ba} = 0.6\frac{\pi}{4}\left(\Delta P_{ann}OD_{tbg}^2 - \Delta P_{tbg}ID_{tbg}^2\right)$$

Corrosion

Reduces metal thickness which leads to a reduction in collapse, burst, and tensile forces

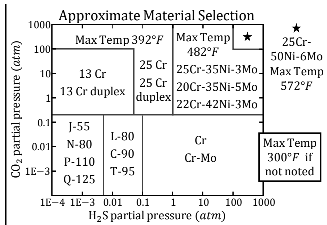
Corrosion Influences

In general, hard metal with large σ_y will corrode quicker than softer metal with a smaller σ_y

Increases Corrosion Rate

- Following variables increase
  - Salt concentration
  - Dissolved gas
  - Dissolved CO_2
  - Dissolved H_2S
  - Temperature
- Decrease in pH

Approximate Material Selection



$$P_{CO_2} = z_{CO_2}P_{BH} \qquad P_{H_2S} = P_{BH}(H_2S\ ppm/1x10^6)$$

z_{CO_2} = CO_2 fraction P_{BH} = static bottom hole P[=]atm

Monitoring – Corrosion Coupon

Corrosion Rate

$$CR = \frac{\Delta mass}{\rho_c A_s \Delta t}[=]\frac{mil}{yr} \qquad \frac{mil}{yr} = \frac{inch}{1000\ yr}$$

CR[=] $thickness\ loss/time$

Pitting Rate

$$PR = d/\Delta t[=]mil/yr$$

d = deepest pit depth Δt = test duration

CR (mil/yr)	PR (mil/yr)	Severity
<1.0	<5.0	Low
1.0–4.9	5.0–7.9	Moderate
5.0–10.0	8.0–15.0	Severe
>10.0	>15.0	Very Severe

Scale

Calcium Carbonate Scale

Calculate the Langelier Saturation Index (LSI) to predict scale

pH_s = pH at saturation $LSI = pH - pH_s$ pH = actual pH

$$pH_s = 0.1\log_{10}(TDS) - 13.12\log_{10}(T) - \log_{10}\{(Hard)(Alk)\} + 44.15$$

TDS = total dissolved solids[=] mg/L T = temperature[=]K

$$Hard = \text{hardness} = 1000[MW_{CaCO_3}([Ca] + [Mg])][=]mg/L$$

Alk = alkalinity[=] mg/L $MW_{CaCO_3} = 100.09\ g/mol$ $[ion]$[=] mol/L

$$Alk = 500[MW_{CaCO_3}([HCO_3] + 2[CO_3] + [OH] + [H])]$$

$[H]$ is negligible

Scale is likely to form if $LSI \ge 0$

Sulfate Scale Tendencies

Calculate the solubility (S) of the ion to predict scale

$$S = \sqrt{X^2 + 4K_{sp}} - X \qquad X = [C] - [A][=]mol/L$$

S = ion solubility[=] mol/L $[C]$ = cation molarity
K_{sp} = equilibrium coefficient $[A]$ = anion molarity

Compute equivalents per liter for cation and anion
Compare solubility to minimum value

$$\frac{(ion\ mg/L)(ion\ charge)}{1000(MW_{ion})}[=]\frac{eq}{L}$$

Scale is likely to form if $S \le eq/L$

Pipeline Design

General Design Steps

1. Determine **proper regulatory policy**
   - Do not proceed until established
   - Design engineer's responsibility
2. Determine pipe diameter
   - Based on flow rate and exit pressure
3. Determine $MAOP$ for pipeline
   - Maximum allowed operating pressure
4. Estimate pipe σ_y and t

$$1.5(MAOP) = \frac{2\sigma_y t}{OD}FET$$

5. Calculate P_i and P_{MT}

$$P_i = \frac{2(SF)\sigma_y t}{OD} \qquad SF = 0.875$$

$$P_{MT} \cong 0.8P_i \qquad P_i\ [=]\ P_b$$

6. Determine test pressure

$$P_d = \frac{2\sigma_y t}{OD}FET$$

$$P_{test} = 1.5MAOP\ or\ 1.25P_d$$ whichever is greater

7. Final burst rating check
   - As specified by correct regulatory policy

P_d and $P_{test} < 0.85P_{MT}$

Example using ASME B31.8

Construction Type	F
Private right of way (PROW)	0.72
PROW on fringe of populated areas	0.6
Sparsely populated residential areas	0.5
Populated areas and public roads	0.4

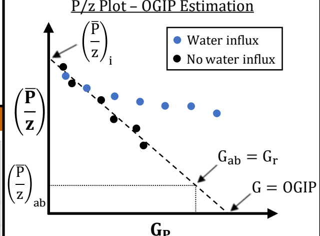
Joint Type	E
Threaded seamless	1.0
Threaded ERW	1.0
Furnace lap welded	0.8
Furnace butt welded	0.6

Flowing Temp	T
< 250 °F	1.0
250 – 300 °F	0.967
300 – 350 °F	0.933
350 – 400 °F	0.900
400 – 450 °F	0.867

Predicting Gas Production

- Gas decline curves are harder to predict due to the high expansivity of gas
  - Can lead to an overestimation of total recovery
- Can use a P/z plot as another predictor
  - Developed from the Real Gas Law

P/z Plot – OGIP Estimation



$$\left(\frac{\bar{P}}{z}\right) = \left(\frac{\bar{P}}{z}\right)_i - \left[\left(\frac{\bar{P}}{z}\right)_i\frac{1}{G}\right]G_P$$

intercept slope

$$RF = \frac{G_r}{G}$$

Recovery factor

Flow Assurance

Common Issues

Damage Type	Detection Methods	Prevention Methods	Removal Methods
Calcium Carbonate Scale	Water analysis Physical sample	Scale inhibitor Scale squeeze	HCl acid job
Barium Sulfate Scale	Water analysis Physical sample	Scale inhibitor	Mechanical removal Re-perforation
Sodium Chloride	Water analysis Physical sample	Reduce pressure drop to reduce gas cooling	Fresh H_2O circulation Re-perforation
Emulsions and Sludge	Physical sample Lab analysis	Emulsion breaker	Emulsion breaker Mutual Solvent
Liquid Block Gas Well	Well history Lab analysis	Limit pressure drop at wellbore	Mutual solvents
Asphaltenes	Physical sample Oil analysis	Inhibitors Application of heat	Inhibitors Application of heat
Paraffin	Physical sample Oil analysis	Inhibitors Application of heat	Inhibitors Application of heat
Formation Fines	Physical sample	Limit production rate Gravel/frac pack	Re-perforation Small frac job
Clay Swelling	Lab analysis Production rate drop	Don't introduce incompatible water	Re-perforation Small frac job
Bacteria	Physical sample Lab culture	Don't introduce bacteria laden water	Bacteriacides

Artificial Lift

Common Issue	Rod Pump	ESP	Gas Lift
Sand	Fair	Fair	Excellent
Paraffin	Poor	Good	Poor
High GOR	Fair	Fair	Excellent
Deviated Hole	Poor	Fair	Good
Corrosion	Good	Fair	Fair
High Volume	Poor	Excellent	Good
Depth	Fair	Fair	Good
Simple Design	Yes	Yes	No
Casing Size	Fair	Good	Good
Flexibility	Fair	Poor	Good
Production Scale	Good	Poor	Fair
Onshore Usage	84%	2%	11%

Gas Lift much higher % offshore

Beam Lift

Pressure Differential Across Plunger

$$\Delta P = PDP - PIP + \Delta P_{SV} \qquad PIP = P_{wf}$$

$$PDP = \nabla P_{tbg}D_{pump} + P_{s,tbg} + P_{fric}$$

Pump Displacement

$$q = 0.1166Nd_p^2S_p\eta_p[=]\ bbl/day$$

N = pump speed[=]spm η_p = pump efficiency
d_p = plunger diameter[=]in

Effective Stroke Length

$$S_p = S + s_{po} - s_{tbg} - s_{rod}[=]in$$

s_{po} = plunger overtravel s_{tbg} = 0 tbg anchored

Rod Loads

$$PPRL = W_{rf} + F_o + W_{D,up}$$

$$MPRL = W_{rf} - W_{D,down}$$

$$W_{rf} = W_{rod}\left(1 - \frac{\rho_f}{\rho_{rod}}\right)$$

W_{rf} = buoyed rod weight

$$SV_{load} = W_{rf} \qquad TV_{load} = F_o + SV_{load}$$

Pump Slippage (Patterson, et al, 2007)

$$q_s = (1 + 0.14N)453\frac{d_p\Delta Pc_p^{1.52}}{L_p\mu_f}[=]\frac{bbl}{day}$$

c_p = plunger clearance[=]in μ_f[=]cp

Resource Economics

Reserve Classification – Common Acronyms

1. PDP: Proved Developed Producing – well is online and producing
2. PDNP: Proved Dev. Non-Producing – reserves are behind pipe, well is shut-in, or waiting on necessary equipment installation to produce
3. PUD: Proved Undeveloped – offsetting wells or existing wells that would require a major recompletion

Time Value of Money

PV = present value
FV = future value
DF = discount factor

$$PV = (FV)(DF)$$

$$DF = \left(1 + \frac{DR}{n}\right)^{-tn}$$

DR = discount rate
DR[=] $decimal/yr$
t = time in years
n = discounting periods per year

Economic Limit

$OPEX$[=] $\$/time$
WI = working interest
NP = net price

$$EL = \frac{OPEX}{NP}\left(\frac{WI}{NRI}\right)[=]\frac{prod}{time}$$

$$NP_o = OP(1 - ST_o - AVT) + GP(GOR)(1 - ST_g - AVT)$$

$$NP_g = GP(1 - ST_g - AVT) + OP(OY)(1 - ST_o - AVT)$$

NP_o = net oil price ST = severance tax OY = oil yield

Converting Production into Cash Flow

Net Revenue	Tax	Cash Flow	Disc. Cash Flow
$(GR)(NRI)$	$(NR)(ST + AVT)$	$NR - OPEX - TAX$	$(CF)(DF)$

GR = gross revenue = (production)(gross price)

Evaluating Potential Investments

Disc. Return on Investment

$$DROI = \frac{DCF}{Investment}$$

Disc. Rate of Return: Discount rate that yields a net present value of zero

Undiscounted Payout: Time required to return initial investment using undiscounted cash flow

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